GRANGER NETWORK ON SANTA MARIA DEL FIORE DOME

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ABSTRACT: The paper investigates the dynamic relationships between cracks and environmental variables, including temperature, humidity, and seismic activity, in the Santa Maria del Fiore Dome. Using Vector Autoregression (VAR) models and Granger causality tests, the study aims to understand the response of cracks to shocks on neighboring cracks. [*]

KEYWORDS: architectural heritage preservation, monument monitoring system, vector autoregressive model, Granger causality, impulse response functions

1 Introduction

The Santa Maria del Fiore Dome is a masterpiece of engineering and a symbol of Florence, Italy. Filippo Brunelleschi's design was revolutionary, and his innovative approach to construction enabled the Dome to be built without any scaffolding or temporary support structures. However, the first cracks on the Dome appeared soon after its construction in the 15th century and have progressively increased, giving rise to concerns about the stability of the monument (Ottoni & Blasi, 2015, Bertaccini, 2015, Bertaccini *et al.* , 2020). To address this issue, a monitoring system consisting of over 160 instruments was installed in the Dome starting from 1955. The present study is part of a long-term project aimed at monitoring the stability of the monument and predicting its future response to distressing phenomena. The objective of this work is to investigate the dynamic relationships between the cracks and the influence of environmental variables. To this aim, Vector Autoregression (VAR) models, Granger causality tests and Impulse Response Functions (IRF) are employed. The paper is structured as follows: Section 2 presents the data and the methodology used for the analysis; Section 3 describes the results.

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2 Data & Methodology

The data, provided by the Opera di Santa Maria del Fiore, consists in daily recordings of cracks width performed by the electronic system installed on the 8 webs of Brunelleschi's Dome by the ISMES (Istituto Sperimentale Modelli E Strutture) in 1987. In this analysis, we focus on the 13 deformometers located on web 4 and we restrict our attention to the period from January 1, 2001 to February 28, 2017. [†] Beside the wall temperature, the data have been supplemented with weather information such as air temperature and humidity, as well as information on earthquakes that occurred within a 50km radius of Florence during the analysis period.[‡]. Given the "breathing" mechanism of the Dome, we suspect that neighboring cracks could affect each other. To investigate the dynamic relationships between cracks as well as the influence of exogenous regressors, we fit the following VARX(p) model (Lütkepohl, 2005),

$$\Phi_p(L)Y_t = c + B_j(L)X_t + \varepsilon_t \tag{1}$$

where: $Y_t = (DF401, \dots, DF413)$ is a vector containing the web crack measurements of all deformometers located on web 4; X_t is a vector of explanatory variables (namely, wall temperature, daily variation of air temperature, humidity and two dummy variables of earthquakes strength); c is a constant term; $\Phi_p(L) = I - \Phi_1 L - \cdots - \Phi_p L^p$ and $B_j(L) = B_0 + B_1 L + \cdots + B_j L^j$ are matrix polynomials in the lag operator L; $\Phi_1, \dots, \Phi_p$ and $B_0, \dots, B_j$ are coefficient matrices for lags 1 to p; and $\varepsilon_t \sim (0, \Sigma)$ is a multivariate white noise.

3 Results

All the variables included in the analysis show a yearly seasonal pattern that is highly persistent over time and there are some deformometers exhibiting non-linear trends (e.g., DF404). Before incorporating time series into a VAR model, they must be made stationary. To achieve this, Fourier terms are used

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[†] This choice is motivated by empirical facts: major cracks are concentrated on the even webs, web 4 and 6 in particular (Ottoni & Blasi, 2015); moreover, early measurements evidence irregular patterns in the web crack evolution that may be due to the instruments' break-in period.

[‡] The city is located close to two fault lines, namely Mugello's composite seismogenic source and the (debated) Prato-Fiesole fault system. Earthquake data was sourced from the website of the Italian National Institute of Geophysics and Volcanology (INGV) and the map of fault lines can be found at `https://diss.ingv.it/diss330/dissmap.html`. Historical recordings of weather information for the city of Florence were obtained from the website "Il meteo", `https://www.ilmeteo.it/meteo/Firenze`

to remove the yearly seasonal pattern, and non-linear trends are removed with a natural cubic spline. This results in the estimation of the following model,

$$Y_t = c + g(t) + \beta_1 sin(2\pi t/365) + \beta_2 cos(2\pi t/365) + \nu_t \tag{2}$$

where: c is the intercept; β_1 and β_2 are the coefficients of the Fourier terms, representing the magnitude and the phase of the seasonal patterns; $g(t)$ is the natural cubic spline function capturing the non-linear trend in the data; and ν_t is the error term. After fitting model (2) separately to each variable, residuals are retained, as they represent the de-seasonalized and de-trended versions of the time series. To provide a snapshot of the results, Figure 1 plots the evolution of DF406 before and after the transformation, showing also the comparison between original and fitted values. The results of the VARX fit (available upon request) evidence that all the exogenous variables included in the model are significant predictors of the web cracks evolution in web 4. In particular, the lagged air temperature variation and lagged wall temperature are generally associated with a reduction in the crack width, whereas lagged humidity and the earthquakes are associated with an increase. Based on the VARX model above, it is also possible to explore the dynamic relationships between the cracks employing Granger causality by testing the pairwise combinations of the deformometers. The resulting Granger network is displayed in Figure 2.[§] Interestingly, DF404 seems the main driver for the evolution of several web cracks and its own dynamics does not appear to be Granger-caused by any other web crack; on the contrary, DF401 appears to be driven by several web cracks and doesn't seem to exert any impact on others. Finally, DF409 seems to be substantially isolated from the remaining deformometers. These results are supported by the IRFs, for which we report a snapshot in Figure 3 below.

[§] For ease of interpretation, the graph only shows unidirectional relationships, i.e., a directed edge is drawn from Y_1 to Y_2 only if past lags of Y_1 predict future values of Y_2 and not the reverse.

Figure 1. *Starting from the left: i) original vs. fitted values; iii) residuals.*

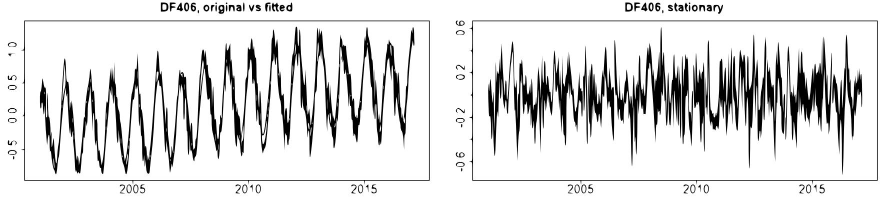


Figure 2. *Graphical representation of the Granger causality network originating from model (1).*

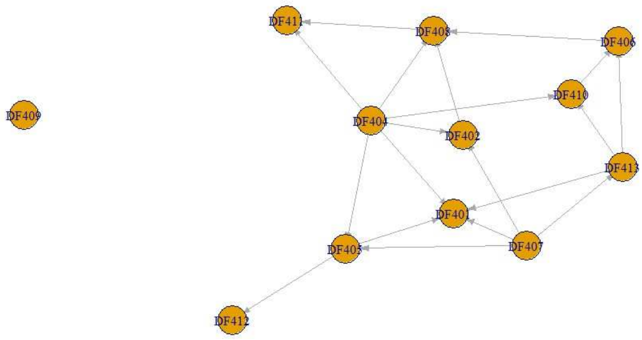
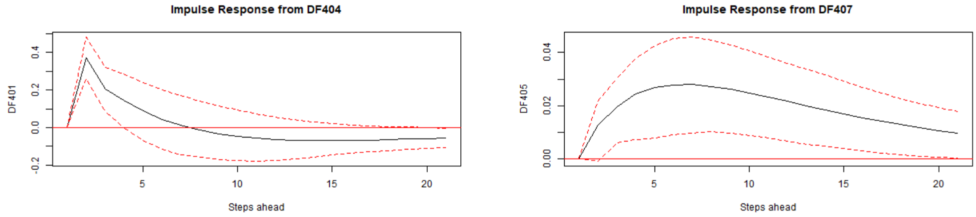


Figure 3. *IRFs for (a) a shock on DF401 from DF404 and (b) a shock on DF405 from DF407. Dashed lines indicate 95% bootstrap-based confidence intervals.*


(a)
(b)

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