DEEP NEURAL NETWORK IN THE MODELING OF THE DEPENDENCE STRUCTURE IN RISK AGGREGATION

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ABSTRACT: We model the dependency structure in the premium and reserve risk sub-module determining the Solvency Capital Requirement (SCR) and the diversification effect (DE). We use the Deep Neural Network (DNN) to estimate marginal distributions modeling the premium and reserve risk of non-life insurance segments, and a copula defining the multidimensional dependency between segments. We use the energy distance to evaluate the error of fitting the copula to the real data. The determined DE when modeling dependencies using the copula method estimated by the use of DNN is compared with DE when modeling dependencies using the method proposed in the Solvency II Directive and using C-vine copulas. The obtained test results indicate that the use of DNN allows for more accurate modeling of the dependency structure, and the determined DE is at the appropriate level.

KEYWORDS: deep neural network, C-vine copulas, Solvency Capital Requirement, diversification effect, risk aggregation.

1 Introduction

Pursuant to the Solvency II Directive (CDR, 2016), each insurance company is obliged to meet the SCR, which is determined by aggregating the risk factors to which the insurance company is exposed. The Directive provides a Standard Formula (SF) where the variance-covariance method is used for risk aggregation and the risk factor correlation matrix is predetermined in the Directive. An alternative to SF are the internal models that better reflect the insurer's business profile. EIOPA (2020) launched a pan-European benchmarking study on diversification in internal models. The aim is to better understand the relationship between dependency modeling and risk aggregation and their diversification benefits. Our dependency modeling method is a proposal for use in internal models. Studies on the impact of the dependency between aggregated risks on the estimated SCR have been conducted by Bermúdez et al. (2013), Cifuentes and Charlin (2016), Mittnik (2020), and Szczęsny (2022a). Eling and Jung (2020) and Szczęsny (2022b) model the structure of dependencies using C-vine copulas introduced in (Bedford and Cooke, 2002). The disadvantages of this approach are discussed by Acar et al. (2012) and Haff (2013). In the literature, the use of DNN for dependency modeling can be found, among others, in Sun et al., (2019), Hassan and Abraham, (2016), and Yunos et al., (2016). We conduct research on real data obtained from Solvency and Financial Condition Reports (SFCRs) of Polish property insurers. We model dependencies based on C-vine copulas (as in (Szczęsny, 2022b)) and a copula fitted to real data using DNN (We generalize the method proposed by (Zeng and Wang, 2022) to determine the four-dimensional distribution). We assess the accuracy of fitted models to real data based on energy distance (Gneiting and Raftery, 2007).

2 Methodology

The solvency requirement for premium and reserve risk in non-life insurance is not greater than the sum of capitals needed to hedge against the risk of each segment separately. The resulting difference is called the Diversification Effect (CDR, 2016). The size of this effect is assessed using the diversification ratio depending on the capital requirements for each segment and the solvency requirement for premium and reserve risk. The key issue in assessing the diversification effect at the appropriate level is both the selection of methods to determine the capital requirement for the L_i risk of individual segments and for the aggregated risk variable L, which depends mainly on the selection of the aggregation function ψ. In the standard formula, it is assumed that L is the sum of dependent random variables L_i with normal distributions with parameters determined by standard deviation for individual segments, where the dependency between L_i is described by the correlation matrix (variance-covariance aggregation is used). It is assumed that the risk variable L has a normal distribution, which usually does not reflect reality.

We estimate the L_i marginal distributions for selected segments and copulas in two ways: using a parametric approach by means of a C-vine copulas as in the paper (Szczęsny 2022b) and innovatively using DNN. Having one-dimensional marginal distributions and the copula function, we invoke Sklar's theorem and combine the determined marginal distributions into a four-dimensional distribution.

3 Empirical result

Due to the limited availability of complete data, we select segments C0020 (insurance against loss of income), C0040 (motor liability insurance), C0050 (other motor insurance), and C0070 (fire and other property damage insurance) for the analysis. We model the risk of these segments using complex factors L_i (i=1,..,4) (Schubert, Grießmann 2007).

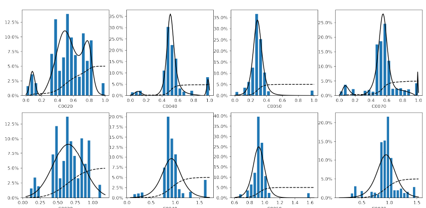
Table 1 presents the actual linear correlation matrix determined differs from the matrix given in the directive (CDR, AnnexIV).

Table 1. Estimated correlations between segments

	C0020	*C0040*	*C0050*	*C0070*
C0020	1	0.2286	0.0034	0.3580
C0040	0.2286	1	0.3028	0.8171
C0050	0.0034	0.3028	1	0.0378
C0070	0.3580	0.8171	0.0378	1

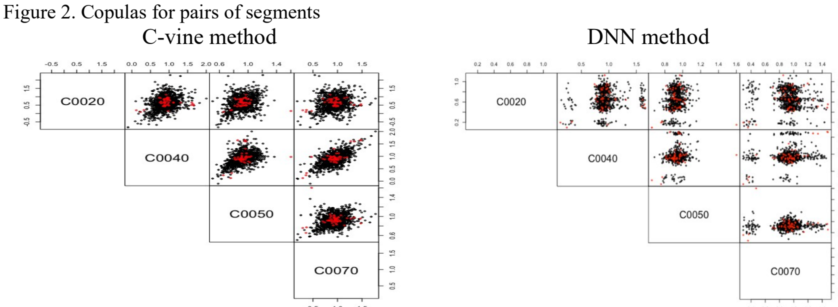
We start the study by estimating the marginal distributions L_i in the two previously given ways. The results for individual segments are presented in Figure 1. Thanks to the method in which we use DNN to estimate probability distributions, we obtain a better fit to real data than in the parametric method. (The upper part of Figure 1.).

Figure 1. Estimated marginal distributions



Next, we model the dependency structure between the segments. Figure 2 presents charts for pairs of segments. Red points represent pairs of real observations and black points are pairs of realizations drawn from the four-dimensional joint distribution.

Figure 2. Copulas for pairs of segments



Having determined the density distributions for individual segments and the distribution of the copula density, we determine the density of the four-dimensional distribution. Then, after estimating four-dimensional distributions in two ways, using energy distance, we evaluate which one better describes the real data. This distance between each vector of actual observations and the realization vectors drawn from the estimated distributions for the distribution determined using DNN is 0.256471 and for the distribution determined using the parametric approach it is 0.2589487. Which indicates a more accurate match using the DNN method.

Finally, we determine the diversification effects obtained in three ways: the first in accordance with the SF contained in the directive, the second based on the results obtained by applying the C-vine copulas and the third by using the DNN. For the three methods, we get the following values: 0.31, 0.35, 0.53. In the analyzed case, a more accurate determination of the dependency structure through the use of a copula causes the value of DE to be higher than the value of DE obtained by the variance-covariance method in the Solvency II Directive.

The obtained results show that the proposed deep neural network architecture is a good candidate for the estimation of marginal distributions as well as the identification of the copula used to describe the structure of dependencies between risk types in internal models.

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